E5B Time constants and phase relationships: R/L/C time constants: definition; time constants in RL and RC circuits; phase angle between voltage and current; phase angles of series and parallel circuits

Time Constant

If a capacitor is connected directly to a voltage source, it will charge up very quickly, almost instantly, to the source voltage. If it is then short-circuited, it will discharge very quickly.

Exponential charging rate

More typically, a capacitor is permitted to charge or discharge though a resistance, in which case the charge or discharge current is limited by the resistor, so the charge or discharge time will be a finite, non-instant time.

You don't need to know this for the exam, but the charge curve is exponential-- it's according to the equation:

$$V_t = E(1 - e^{-1/RC})$$

where
t is the charging time in seconds
V_t is the capacitor voltage at time t
E is the source voltage
e is the natural logarithmic base, 2.718
R is the resistance in ohms
C is the capacitance in farads.

In case you're not mathematically inclined, let's say this about an exponential charging curve: the more the capacitor getscharged, the slower the charge increases. When you first connect E, the charge is zero, so current flows in as fast as the resistor permits. After some small time increment, the capacitor has become slightly charged by the charging current, and that charge opposes the flow of further charging current, so you charge a little slower, and so on-- the more charged you get, the more you oppose charge current, so the slower you charge.

Time Constant = RC

In practical circuit work, it can be handy to have a way to get a “handle” or a "ballpark figure” on how long it will take some circuit to reach an appreciable level of charge.

(This is something you DO need to know for the exam:) The above equation dictates that the capacitor voltage V_t will reach 63.2% of voltage E (the source voltage) in R (in ohms) times C (in farads) seconds. (That's how the math works out.) Thus R*C is known as the time constant.
A capacitor in an RC circuit will be charged to 63.2% of the supply voltage in one time constant. (E5B01)

Similarly, if an RC network containing a charged capacitor is shorted, the capacitor will DIScharge by 63.2% in one time constant-- that is, it will discharge to 36.8% of its initial value of stored charge in one time constant. (E5B02).

To analyze the charge or discharge time of an RC circuit over several time constants, just take the initial charge of each time constant to be the ending charge after the previous time constant. For example, in one time constant the capacitor discharges to 36.8 of the initial value, and in the second time constant it discharges to 36.8 %of that, or 36.8% OF 36.8% of the initial value. 0.368 * 0.368 = 0.135 = 13.5%. So after two time constants the capacitor is discharged to 13.5% of the starting value. (E5B03)

If a circuit has two 220-microfarad capacitors and two 1 megohm resistors all in parallel, we must first simplify or resolve it into a circuit with one R and one C.

You'll recall from Technician- and General-class license exams that when capacitors are in parallel, the values simply add. So the circuit resolves to one capacitive leg containing 440 microfarads of capacitance.

And you'll recall that when two equal resistors are in parallel, the effective resistance is half the value of either resistor, so the circuit resolves to one resistive leg containing a resistance of 0.5 Megohms.

So RC, the time constant of such a circuit, is

$440 * 10^{-6} * 0.5 * 10^{6} = 220 * 10^{0} = 220$ seconds. (E5B04) (Any number raised to the zero power = 1)

Or if you were asked “How long does it take for an initial charge of 20 V DC to decrease to 7.36 V DC”, you might note that this is asking you how long it takes for the charge to decrease to 36.8% of its initial value-- or in other words, how long is one time constant.

If you were asked this in regard to a circuit comprising a .01-microfarad capacitor and a 2-megohm resistor, one time constant = $0.01 * 10^{-6} * 2 * 10^{6} = 0.02 * 10^{0} = 0.02$ seconds. (E5B05)

Or if you were asked “How long does it take for an initial charge of 800 V DC to decrease to 294 V DC”, you might note that this is asking you how long it takes for the charge to decrease to 36.8% of its initial value-- or in other words, how long is one time constant for this circuit.

If you were asked this in regard to a circuit comprising a 450-microfarad capacitor and a 1-megohm resistor, one time constant = $450 * 10^{-6} * 1 * 10^{6} = 450 * 10^{0} = 450$ seconds. (E5B06)

Phase Angle

In general, calculating phase angle when given R, L, and C values requires the use of trigonometry, with which many of us are not familiar. **However**, the component values and multiple-choice answers provided on the exam enable the right answer to be determined “by inspection”, as mathematicians say-- once you learn how to diagram the values in a question, you'll be able to see that for some questions the phase angle must be considerably less than 45 degrees, or perhaps for some questions considerably more, and you'll be able to select the right answer accordingly.

Vectors

Let's say first that there are two kinds of quantities—*scalar* quantities, and *vector* quantities

- **Vector quantities** have magnitude *and direction*. For example, if you say that you're driving your car at **30** miles per hour, at any given instant you're heading in some specific *direction* at that magnitude of speed.
- **Scalar quantities** have magnitude only. For example, if you say that you have **30** dollars in your wallet, "direction" does not apply.

We'll need to think in terms of vector quantities to analyze the behavior of RLC circuits since the L's and C's exhibit voltages, and currents that are "out of phase" with each other and with the R's.

A vector quantity is diagrammed on a graph as a line with an arrowhead on one end; the *length* of the line represents the magnitude of the quantity, and the *direction* of the line indicates the direction of the quantity. Such a line is simply called a vector.

When diagramming R's, L's, and C's on a graph with the usual rectangular coordinates,

- resistance is graphed "to the right" along the X axis
- inductive reactance is graphed "up" on the Y axis
- capacitive reactance is graphed "down" on the Y axis

Then the **impedance** Z of the circuit is represented by a vector drawn from the origin of the graph (coordinates 0,0) to the end of the "vector trail" of R, L, and C.

For example, in an RLC circuit where R=1 kilohm (1000 ohms), X_C = 500 ohms, and X_L = 250 ohms, the vector diagram is as shown in Fig. 1.

- From the origin at 0,0, a vector R is drawn to 1000 on the X axis (to 1000,0),
- From there a vector X_L is drawn straight up to a Y value of 250 (to 1000,250),
- From there a vector X_C is drawn straight down for 500 units, and thus it ends at -250 on the Y axis, at coordinates 1000,-250.

(The three vectors can be drawn in any order, and the "trail" for these values will always end at coordinates 1000,-250.) The impedance, Z, is represented by a vector drawn from the origin (0,0) to 1000,-250. Although the exam doesn't ask for the value of the impedance Z, it could be determined by geometry

Pythagorus' theorem: "The square of the hypotenuse equals the sum of the squares of the two sides"

sqrt($1000^2 + 250^2$) = 1031 ohms.

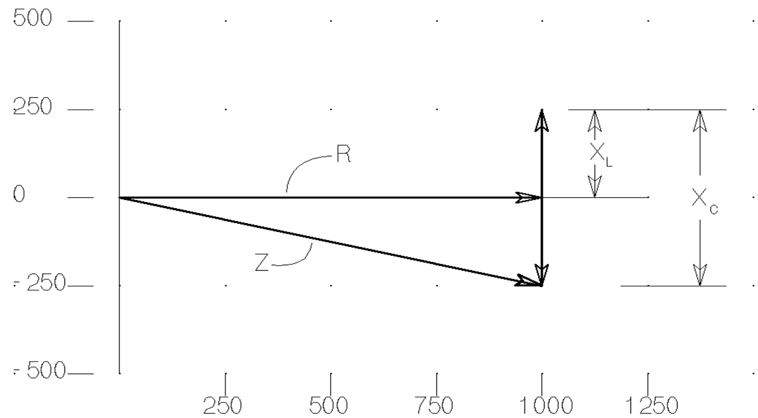

Fig. 1

The phase angle. the angle between Z and R, can be determined by trigonometry as $\sin^{-1}$ (-250/1031). But not all of us know trigonometry. **However**, among the multiple-choice answers we find answers of either 68.2 degrees or 14.0 degrees. We can see *by inspection* of the graph that the angle between Z and R must be a "sharp" angle, well below 45 degrees, so 68.2 degrees can't be the right answer. And, since the effective reactance of the circuit is capacitive (500 ohms of X_C more than cancels out the 250 ohms of X_L) we know from our old friend ELI the ICEman that voltage lags current in a capacitive reactance . (E5B09). So, the phase angle is 14.0 degrees with the voltage lagging the current. (E5B07)

Or if R = 100 ohms, X_L = 75 ohms, and if X_C = 100 ohms, the vector diagram could be as shown in Fig. 2. Again it is seen that the phase angle must be a sharp angle,so the multiple choice answers containing 76 degrees can't be right, so one of the 14-degree answers must be right. Again the effective reactance is capacitive, so again the voltage lags the current (and the phase angle is 14 degrees). (E5B08)

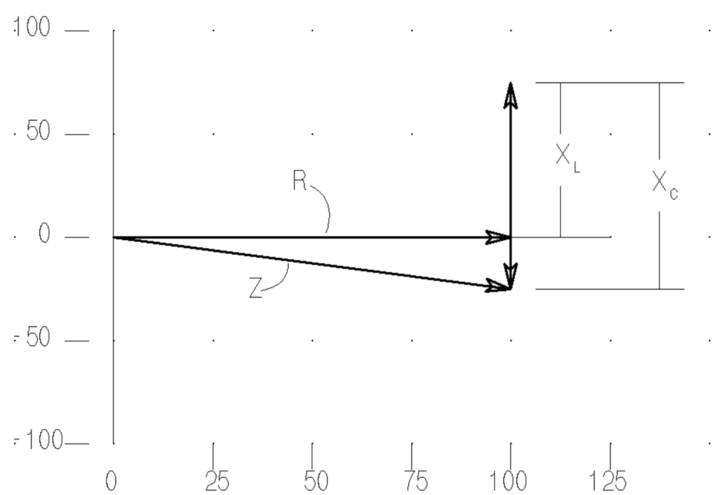

Fig. 2

Or if R = 100 ohms, X_C = 25 ohms, and X_L = 50 ohms,

- We could draw vector R from 0,0 to 100,0
- We could draw vector X_C down to 100,-25
- We would draw vector X_L up to 100,25
- Now we can draw vector Z from 0,0 to 100,25

(The drawing is, as the old saying goes, "left as an exercise for the serious student".)
Again, we see that the angle must be a sharp one, so the 76-degree choices can't work, so one of the 14-degree answers must be right. Now the effective reactance is inductive, so the phase angle is 14 degrees with the voltage leading the current. (E5B11) The voltage leads the current by 90 degrees in an inductor. (E5B10)

Or if R = 100 ohms, X_C = 75 ohms, and X_L = 50 ohms,

- We could draw vector R from 0,0 to 100,0
- We could draw vector X_C down to 100,-75
- We would draw vector X_L up to 100,-25
- Now we can draw vector Z from 0,0 to 100,-25

Again, we see that the angle must be a sharp one, so the 76-degree choices won't work, and one of the 14-degree choices must be right. And the effective reactance is capacitive, so the phase angle is 14 degrees with the voltage lagging the current. (E5B12)

Or if R = 1000 ohms, X_C = 250 ohms, and X_L = 500 ohms,

- We could draw vector R from 0,0 to 1000,0
- We could draw vector X_C down to 1000,-250
- We could draw vector X_L up to 1000,250
- Now we can draw vector Z from 0,0 to 1000,250

We see that the angle must be a sharp one, so the 81.47-degree answers won't work and the right answer must be one of the 14.04 degree answers. And, the effective reactance is inductive, so the phase angle is 14.04 degrees with the voltage leading the current. (E5B13)