E5A Resonance and Q: characteristics of resonant circuits: series and parallel resonance; Q; half-power bandwidth; phase relationships in reactive circuits

Inductors and Capacitors

We're able to ***tune*** our receivers and transmitters by means of a property that occurs in LC circuits (circuits containing inductors "L" and capacitors "C")-- a property called ***resonance***.

Phase

You'll now need to recall the word *phase* and its implications. In a resistance, the voltage and the current are "in phase"-- that is, when the voltage is zero, the current is zero; when the voltage is maximum, the current is maximum. Another way of saying that the voltage and current are in phase is to say that the *phase angle* between them is zero degrees.

"Degrees" here refers to the 360 degrees of a full cycle of the sine wave of an rf current or voltage. In inductors and capacitors, the voltage and the current are OUT OF PHASE by 90 degrees. (90 degrees in practical, approximate terms-- actually attaining 90.000 degrees would happen only in "pure" L's and C's-- L's and C's with zero resistance. This is only theoretical, since any conductive material has some resistance, however minuscule.)

To say that two sine waves (such as a voltage wave and a current wave) are out of phase by 90 degrees is to say that one curve's zero occurs at the time of the other curve's positive or negative maximum, and vice versa.

While a resistor can only PASS energy (or dissipate it as heat), L's and C's can STORE energy for brief periods of time (on the order of the cycle duration of an rf signal). This gives rise to their unique reactive properties.

An inductor stores energy in a magnetic field. In an inductor, the VOLTAGE LEADS the current by 90 degrees. (For purposes of Section E5, you don't need to know WHY this is so-- only THAT it is so.)

A capacitor stores energy in an electric field. In a capacitor, the CURRENT LEADS the voltage by 90 degrees. (Again, you don't need to know why, although it's commendable if you WANT to know why. There are many reference books that can help, including the ARRL Handbook.)

A handy mnemonic (memory aid) for remembering what leads what is
ELI the ICEman E leads I in an L; I leads E in a C.

Reactance

It would also be well at this point to review *reactance*. Inductors exhibit *inductive reactance* X_L; capacitors exhibit *capacitive reactance* X_C.

- Reactance is LIKE resistance in that it limits current.
- Reactance is LIKE resistance in that it's measured in ohms.
- Reactance is UNLIKE resistance in that it does not consume power.
- Reactance is UNLIKE resistance in that the current through a reactance is 90 degrees out of phase with the voltage across it. (In a resistance, they're in phase.)

X_L = 2*pi*f*L (thus it INcreases as f increases)

X_C = 1 / (2*pi*f*C) (thus it DEcreases as f increases)

Since reactance is not in phase with resistance, the ohmic values of reactance and resistance cannot be simply added to determine the overall circuit impedance-- the impedance is the *vector resolution* of the inductive reactance, the capacitive reactance, and the resistance. (See section E5B for more on vector resolution.)

Now, RESONANCE..........

In a circuit with both L and C, there will be some frequency at which the inductive reactance and the capacitive reactance are equal. (Bear in mind that for given values of L and C, inductive reactance INcreases as frequency increases, and capacitive reactance DEcreases as frequency increases.) There is one frequency at which the inductive reactance and capacitive reactance are equal; this is the resonant frequency-- the frequency at which resonance occurs (E5A02).

Series resonance (the L and the C are in series)

The advantage of series resonance is that since the inductive reactance and capacitive reactance are out of phase with each other, when equal they cancel each other out, leaving only the RESISTANCE of the circuit elements to constitute the impedance(E5A03). As the input frequency to the circuit is varied through the resonant frequency, the current through the circuit will vary through its maximum (E5A05). Since there is, in effect, only resistance and no reactance in the circuit at resonance (X_L and X_C cancel each other out), the voltage and the current are in phase (E5A08) as with any pure resistance.
Since the current in a series resonant circuit, limited only by the resistance of the circuit, may be relatively high, the voltages across the L and the C (I*X_L and I*$X_C$respectively) can also be relatively high, even higher than the voltage applied to the circuit (E5A01). This is possible because those voltages cancel each other out within the circuit and do not appear externally to the circuit.

Parallel resonance (the L and the C are in parallel)

Parallel circuits are a little more involved because in real-world circuits the resistance is not symmetrical between the L leg and the C leg. While the plates (or foil layers, or whatever) of a capacitor can be manufactured with extremely low resistance, winding enough wire to realize a sufficient amount of inductance will also introduce some resistance. That resistance didn't complicate a series-resonant circuit because there it's in the path common to both the L and the C.

Let's say first (though it's entirely theoretical) that in a pure (no resistance) parallel LC circuit, current at the resonant frequency would just circulate between the L and the C-- since I_L (the inductor current) and I_C (the capacitor current) are 180 degrees out of phase, they cancel each other out in terms of current flowing external to the circuit. So the external current, that is the output current or the input current, will be at its minimum (E5A07). Therefore, a parallel-resonant circuit presents a high impedance at the resonant frequency, and is usually connected between some signal point and ground, where it will shunt all frequencies to ground except the resonant (desired) frequency. The circulating current will be a maximum at resonance (E5A06) because then there is complete cancellation of external current.

Question E5A04
"What is the magnitude of the impedance of a circuit with a resistor, an inductor and a capacitor all in parallel, at resonance?"

Such a circuit is shown in Fig. 1(B). We should say first that typically one would not construct such a circuit-- it's an *equivalent* circuit for analyzing the real-world circuit shown in Fig. 1(A).

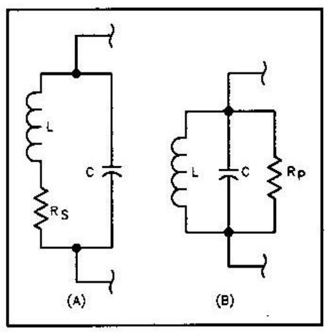

Fig. 1

In a real-world circuit, capacitor C can be manufactured with negligible resistance, but winding enough wire to provide a significant inductance value to inductor L introduces a small but non-negligible resistance represented by R_S.

> [Determining the equivalent circuit (B) from the real-world circuit (A) is not required for the Extra-class exam, and so will not be discussed in detail here. Suffice it to say that resistance R_P typically calculates to a high value; e.g., for X_L of 350 ohms and R_S of 5 ohms, R_P calculates to 24,500 ohms.]

Anyway, for the circuit described in the question (circuit (B)), the impedance is equal to the circuit resistance (E5A04).

And since the L and the C circulate current between them and have no effect on the external current, the current through and the voltage across a parallel-resonant circuit are in phase (E5A09).

Series-resonant circuits, with low impedance at the resonant frequency, are typically connected in series with a signal so that signal must flow *through* them; they pass the resonant frequency and impede other frequencies.

Parallel-resonant circuits, with high impedance at the resonant frequency, are typically connected in shunt (between a signal point and ground). Undesired frequencies are thus “shorted” to ground while signals of the resonant frequency are not affected.

Calculating the Resonant Frequency

For any combination of L and C, we can determine the resonant frequency from the formula:

f = 1 / 2*pi*sqrt(L*C)

where f is in Hz, L is in henrys, and C is in farads.
Since Hz, henrys, and farads are not the commonly used units and are not convenient to use, it is advantageous to transpose the formula to

f = 1000 / 2*pi*sqrt(L*C)

where f is in MHz, L is in microhenrys, and C is in picofarads. (Pi, as always, is 3.1416).
Here are some examples:

If L = 50 microhenrys and C = 40 picofarads,
L*C = 50 * 40 = 2000
sqrt(2000) = 44.7
2 * 3.1416 * 44.7 = 281
f = 1000 / 281 = 3.56 MHz (E5A14)

If L is 40 microhenrys and C is 200 picofarads,
L*C = 40 * 200 = 8000
sqrt(8000) = 89.4
2 * 3.1416 * 89.4 = 561.7
f = 1000 / 561.7 = 1.78 MHz (E5A15).

If L is 50 microhenrys and C is 10 picofarads,
L*C = 50 * 10 = 500
sqrt(500) = 22.4
2 * 3.1416 * 22.4 = 140.4
f = 1000 / 140.4 = 7.12 MHz (E5A16).

If L = 25 microhenrys and C = 10 picofarads,
L*C = 25 * 10 = 250
sqrt(250) = 15.8
2 * 3.1416 * 15.8 = 99.3
f = 1000 / 99.3 = 10.1 MHz (E5A17).

If you worked these with a calculator, you had many more digits of “precision” than these answers reflect. Rounding them off is perfectly acceptable.

You note that the circuit resistance has no role in determining the resonant frequency. It does affect the circuit bandwidth, as we'll now see.

Q (circuit quality)

Q is a widely used “figure of merit” for reactive components, and is simply defined as the reactance X divided by the resistance R. Since both X and R are expressed in ohms, the units cancel out and Q is not expressed in any units, but just as a number. A “pure” reactance (with R of zero) would have infinite Q-- but since everything has some amount of resistance, real-world Q's are finite, though they can be quite high.

Non-zero resistance in a resonant circuit tends to broaden its response, since the resistance provides a path independent of the reactive components. (Sometimes, the circuit designer deliberately introduces resistance to “broadband” the design.)

Half-Power Bandwidth

A useful construct for getting a handle on the effects of resistance on a resonant circuit is the half-power bandwidth, defined as the frequency range over which the output of the circuit is within 3 db (half power) of the maximum (which occurs at the precise resonant frequency). It is calculated simply as the resonant frequency divided by the circuit Q.
Here are some examples:

If the resonant frequency is 1.8 MHz and the Q is 95, the half-power bandwidth is 1800 kHz/95 = 18.9 kHz (E5A10).

If the resonant frequency is 7.1 MHz and the Q is 150, the half-power bandwidth is 7100 kHz/150 = 47.3 kHz (E5A11).

If the resonant frequency is 3.7 MHz and the Q is 110, the half-power bandwidth is 3700 kHz/95 = 31.4 kHz (E5A12).

If the resonant frequency is 14.25 MHz and the Q is 187, the half-power bandwidth is 14250 kHz/187 = 76.2 kHz (E5A13).

